4-3 Videos Guide

4-3a

Theorems (statement and proof):

- The Fundamental Theorem of Calculus, Part 1: If f is continuous on $[a, b]$, then the function $g(x) = \int_a^x f(t)\, dt,\ \ a \le x \le b$ is continuous on $[a, b]$, and $g'(x) = f(x)$. That is, $\frac{d}{dx}\left[\int_a^x f(t)\, dt\right] = f(x)$ (differentiation is the inverse of integration).

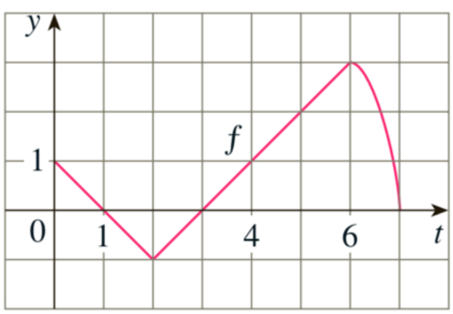
4-3b

- The Fundamental Theorem of Calculus, Part 2: If f is continuous on $[a, b]$, then $\int_a^b f(x)\, dx = F(b) - F(a)$, where F is any antiderivative of f [that is, $F'(x) = f(x)$].

Exercises:

4-3c

- Let $g(x) = \int_0^x f(t)\, dt$, where f is the function whose graph is shown.
  (a) Evaluate $g(x)$ for $x = 0, 1, 2, 3, 4, 5,$ and 6.
  (b) Estimate $g(7)$.
  (c) Where does g have a maximum value? Where does it have a minimum value?
  (d) Sketch a rough graph of g.



4-3d

- Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.
  - $h(u) = \int_0^u \frac{\sqrt{t}}{t+1}\, dt$
  - $h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4+1}\, dz$

  Note: The Chain Rule applies to the derivative, as appropriate:

$$\frac{d}{dx}\left[\int_a^{u(x)} f(t)\, dt\right] = f(u(x))u'(x)$$

4-3d

- Evaluate the integral.
  - $\int_0^1 (1 - 8v^3 + 16v^7)\, dv$
  - $\int_0^4 (4 - t)\sqrt{t}\, dt$
  - $\int_{\pi/4}^{\pi/3} \csc^2\theta\ d\theta$